Models for Circular causality

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Circularity: motivating scenario, Alice and Bob

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2. B says: I ship, after you pay (the action is b)

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logic: $b \rightarrow a$ $\qquad\qquad a \rightarrow b$

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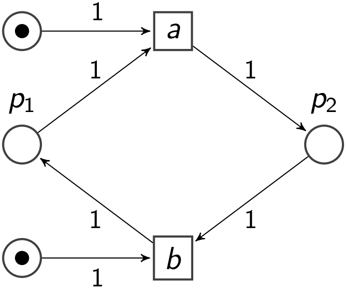

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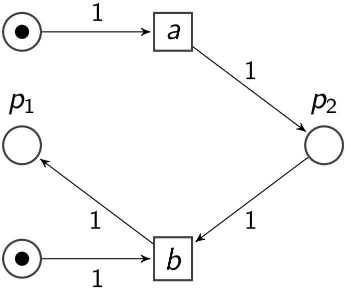

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dependencies are clear and it is possible to move

but what happens to A if B does something else?

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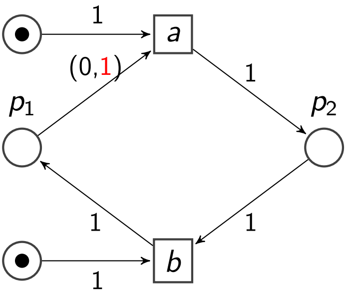

Circularity: motivating scenario, Alice and Bob

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dependencies are clear and there are moves

Circularity: motivating scenario, Alice and Bob

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2. B says: I ship, after you pay

if B does not ship then A may complain

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Simple: exploit new kind of arcs in Petri nets, new dependency relation in event structure and new connectives in logic...

Lending Petri nets

The idea

Rather simple: allow the marking (resources allocated) to be negative

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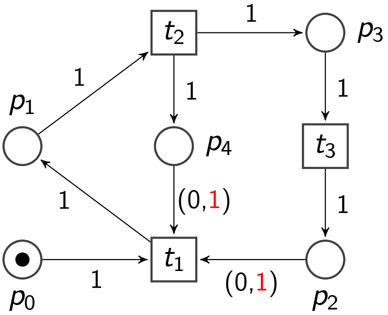

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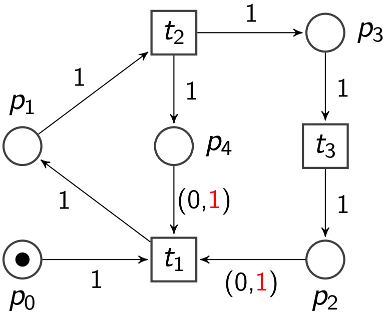

t_1 can be executed lending tokens from p_2 and p_4

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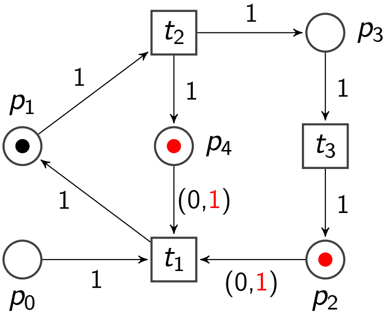

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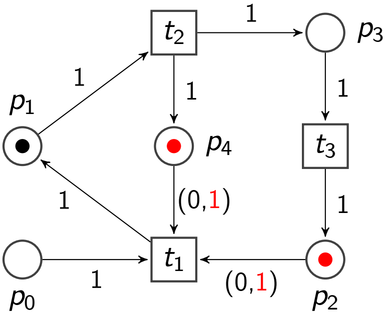

the execution of t_2 gives a token back to p_2

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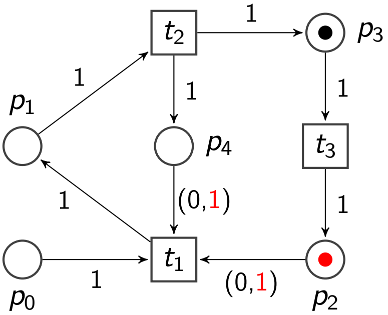

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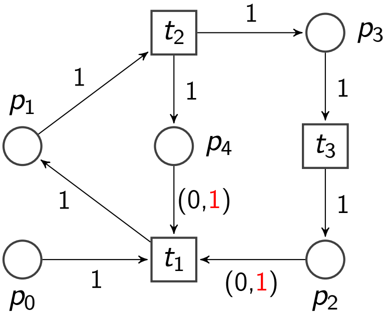

the reached marking is non negative

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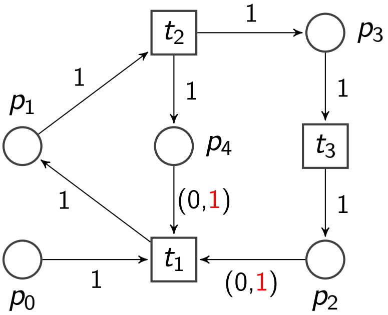

tokens can be lent from places p_4 and p_2 only

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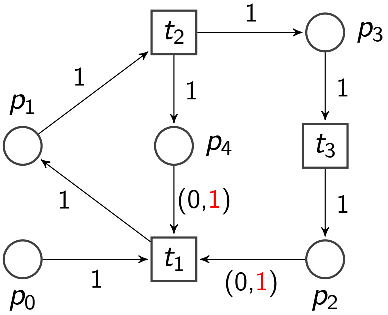
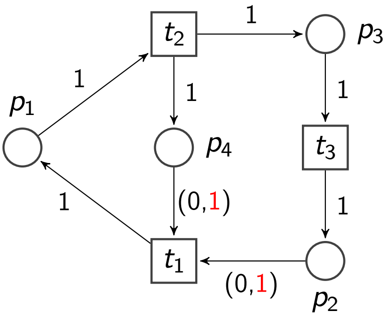


each execution of t_1 may lend a token from p_2 and one from p_4

Lending Petri Net (LPN)



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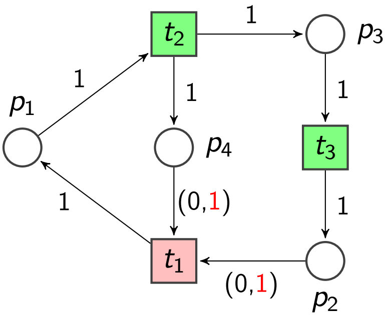

t_1 depends on t_2 and t_3 (they have to produce the tokens t_1 uses...)

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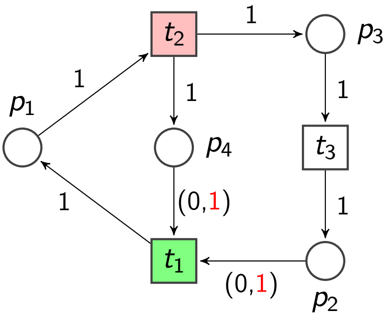

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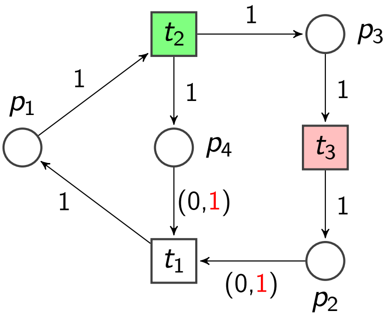
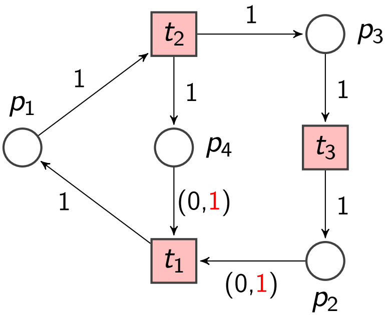


but t_2 depends on t_1 as well... and t_3 depends on t_2

Lending Petri Net (LPN): circularity



Summing up: t_1 depends on t_2 and t_3, t_2 depends on t_1 and t_3 depends on t_2... hence on t_1... $\Longrightarrow$ circularity

Formalizing the intuition

The usual ingredients of a Petri net: $\langle S, T, F, m\rangle$ (places, transitions, weight function and the initial marking)

a labeling over places and transitions (the labels are chosen from a set $\mathcal{L}$ and the labeling mapping may be partial)

a lending function $L : S \times T \to \mathbb{N}$ is added

the requirement that markings are non negative is relaxed: $m : S \to \mathbb{Z}$ (but the initial marking must be non negative)

when a marking m is non negative we say that m is honored

other approaches have debit tokens or antitokens

Lending tokens

A LPN is the tuple $\langle S, T, F, L, \ell, m \rangle$

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$$m'(s) = m(s) - (F(s,t) + L(s,t)) + F(t,s)$$

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if $s \in {}^{\circledcirc}t$ then tokens from s are used (exactly $L(s,t)$), regardless if there are or not: credits

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LPN are a conservative extension of Petri net

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if we allow antitokens and we allow that places may contain tokens and antitokens then the nets are not any longer Turing powerful

Event structure with circular causality

The idea

we introduce a relation among events modeling a dependency on credit

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e_1 depends on the happenings of e_2 and e_3 and may assume that these two events will eventually happen

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in the whole sequence e_1 e_2 e_3 all the events are causally justified

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some ingredients are standard: events (E), conflict relation ($\#$), standard causality ($\vdash$) and we add the new relation $\Vdash$ modeling circular causality

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$\mathcal{E} = (E, \#, \vdash, \Vdash)$

- $\# \subseteq E \times E$ which is symmetric and irreflexive
  and $\mathit{Con} = \{X \subseteq E \mid CF(X)\}$ where CF means conflict free (i.e. $\forall e, e' \in X.\ \neg(e \# e')$)
- $X \circ e$ with $X \in \mathit{Con}$ and $\circ \in \{\vdash, \Vdash\}$ and they are saturated: $X \circ e$ and $X \subseteq Y \in \mathit{Con}$ implies $Y \circ e$

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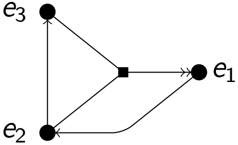
the representation of causality (both standard and circular) we adopt allows for conjunctive or disjunctive causality

Formalizing the intuition

we draw the events and their relation as follows:



and the causality relations are: $\{e_2, e_3\} \Vdash e_1$ and $\{e_1\} \vdash e_2$, $\{e_2\} \vdash e_3$

Configuration

the key notion of event structure: $\mathcal{E} = (E, \#, \vdash)$ (without circular causality)

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a configuration $C \subseteq E$ is a set of events such that

- each subset of C is conflict free
- the events in C can be linearized $\{e_1, \ldots, e_n, \ldots\}$ in such a way that for all $i > 0$, $\{e_1, \ldots, e_{i-1}\} \vdash e_i$ (secured)

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the configurations of an event structure form a family $\mathcal{F}$ which is

1. Coherent: (if a lub is defined then it is a configuration)
2. Finite: $\forall C \in \mathcal{F}.\ \forall e \in C.\ \exists C_0 \in \mathcal{F}.\ e \in C_0 \subseteq_{fin} C$
3. Coincidence free: $\forall C \in \mathcal{F}.\ \forall e, e' \in C.\ (e \neq e' \implies (\exists C' \in \mathcal{F}.\ C' \subseteq C\ \wedge\ (e \in C' \iff e' \notin C'))$

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the configurations of an event structure with circular causality form a quasi-family $\mathcal{F}$ which is

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Coincidence freeness does not hold

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let $\mathcal{F}$ be the quasi-family of configurations obtained by the CES $\mathcal{E} = (E, \#, \vdash, \Vdash)$, define

1. $e \ \hat{\#} \ e' \iff \forall C \in \mathcal{F}.\ e \notin C \ \vee \ e' \notin C$
2. $X \ \hat{\Vdash} \ e \iff CF(X) \wedge X$ is finite $\wedge\ \exists C \in \mathcal{F}.\ e \in C \subseteq X \cup \{e\}$

then $\left(E, \hat{\#}, \emptyset, \hat{\Vdash}\right)$ is an event structure with circular causality such that its configurations are precisely $\mathcal{F}$

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$\{e_2\} \not\in \mathcal{F}$ as it requires that at least e_1 is in it ($\{e_1\} \Vdash e_2$)

Calculating reachable events

For all $X, Y, Z \subseteq E$ of a conflict-free $\mathcal{E}$, define

$$G_Y(Z) = Y \cup \{e \mid Z \vdash e\} \qquad F(X) = \mathrm{lfp}\, G_{\{e \mid X \Vdash e\}}$$

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this gives an algorithm which is polynomial

Propositional Contract Logic

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but $(a \rightarrow b) \wedge (b \rightarrow a) \rightarrow a \wedge b$ is not a theorem and we would like to have this

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it is an extension of IPC and there is no homomorphic encoding of PCL into IPC

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A (part of) a natural deduction system for PCL (Δ is a set of PCL formulae)

$$\frac{}{\Delta, A \vdash A}\ (\textsc{Id}) \qquad \frac{\Delta \vdash A \qquad \Delta \vdash B}{\Delta \vdash A \wedge B}\ (\wedge\mathrm{I}) \qquad \frac{\Delta \vdash A \wedge B}{\Delta \vdash A}\ (\wedge\mathrm{E1}) \qquad \frac{\Delta \vdash A \wedge B}{\Delta \vdash B}\ (\wedge\mathrm{E2})$$

$$\frac{\Delta \vdash A}{\Delta \vdash A \vee B}\ (\vee\mathrm{I1}) \qquad \frac{\Delta \vdash B}{\Delta \vdash A \vee B}\ (\vee\mathrm{I2}) \qquad \frac{\Delta \vdash A \vee B \qquad \Delta, A \vdash r \qquad \Delta, B \vdash r}{\Delta \vdash r}\ (\vee\mathrm{E})$$

$$\frac{\Delta, A \vdash B}{\Delta \vdash A \rightarrow B}\ (\rightarrow\mathrm{I}) \qquad \frac{\Delta \vdash A \rightarrow B \qquad \Delta \vdash A}{\Delta \vdash B}\ (\rightarrow\mathrm{E})$$

$$\frac{\Delta \vdash B}{\Delta \vdash A \twoheadrightarrow B}\ (\twoheadrightarrow\mathrm{I1}) \qquad \frac{\Delta \vdash A \twoheadrightarrow B \qquad \Delta, B \vdash A}{\Delta \vdash B}\ (\twoheadrightarrow\mathrm{E})$$

$$\frac{\Delta \vdash A \twoheadrightarrow B \qquad \begin{array}{l}\Delta, A' \vdash A \\ \Delta, B \vdash A' \twoheadrightarrow B'\end{array}}{\Delta \vdash A' \twoheadrightarrow B'}\ (\twoheadrightarrow\mathrm{I2})$$

Some facts about PCL

binary handshaking:

$\vdash (A \twoheadrightarrow B) \wedge (B \twoheadrightarrow A) \ \rightarrow \ A \wedge B$

multiparty handshaking

$\vdash (A_1 \twoheadrightarrow A_2) \wedge \cdots \wedge (A_{n-1} \twoheadrightarrow A_n) \wedge (A_n \twoheadrightarrow A_1) \ \rightarrow \ A_1 \wedge \cdots \wedge A_n$

contractual implication is stronger than the intuitionistic one

$\vdash (A \twoheadrightarrow B) \rightarrow (A \rightarrow B)$

the converse is false

$\not\vdash (A \rightarrow B) \rightarrow (A \twoheadrightarrow B)$

PCL is decidible

Our example

the theory: $a \wedge b \twoheadrightarrow c$, $c \rightarrow a$ and $a \rightarrow b$

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as you can expect, it is possible to prove

$$a \ \wedge \ b \ \wedge c$$

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a simpler theory: $\Delta = \{a \rightarrow b, b \twoheadrightarrow a\}$

Our example

a simpler theory: $\Delta = \{a \to b, b \twoheadrightarrow a\}$

a proof of a is

$$\dfrac{\Delta \vdash b \twoheadrightarrow a \qquad \dfrac{\Delta, a \vdash a \to b \qquad \dfrac{}{\Delta, a \vdash a}\,(\textsc{Id})}{\Delta, a \vdash b}\,(\to\mathrm{E})}{\Delta \vdash a}\,(\twoheadrightarrow\mathrm{E})$$

Relating these models

CES vs PCL

The idea: encode a CES into a PCL theory and then use the deductive apparatus of PCL

CES vs PCL

consider a conflict-free CES $\mathcal{E} = (E, \vdash, \Vdash)$

an event e is reachable iff there exists a configuration $C \in \mathfrak{F}$ such that $e \in C$

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we associate a PCL theory $([\mathcal{E}])$ as follows

$$\begin{array}{rcl} [(X_i \circ e_i)_{i \in I}] & = & \{[X_i \circ e_i] \mid i \in I\} \\ [X \circ e] & = & \left(\bigwedge X\right) [\circ]\, e \end{array} \qquad where\, [\circ] = \begin{cases} \rightarrow & \text{if } \circ = \vdash \\ \twoheadrightarrow & \text{if } \circ = \Vdash \end{cases}$$

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with this encoding we can prove:

$$e \text{ is reachable in } \mathcal{E} \text{ iff } [\mathcal{E}] \vdash_{\mathsf{PCL}} e$$

$\Rightarrow$ we can use the logic to prove reachability

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$\mathcal{E} = (E, \#, \vdash, \Vdash)$ $\quad !X = \{!e \mid e \in X\}$ $\quad$ (thus $!E \cap E = \emptyset$)

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the encoding:

$$\begin{array}{rcl} [(X_i \circ e_i)_{i\in I}]_{\mathfrak{F}} & = & \{[X_i \circ e_i]_{\mathfrak{F}} \mid i \in I\} \\ [X \circ e]_{\mathfrak{F}} & = & (!e \wedge X \wedge !X)[\circ]\ e \qquad \textit{where}\ [\circ] = \begin{cases} \rightarrow & \text{if } \circ = \vdash \\ \twoheadrightarrow & \text{if } \circ = \Vdash \end{cases} \\ [a \# b]_{\mathfrak{F}} & = & (!a \wedge !b) \rightarrow \bot \end{array}$$

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[a \# b]_{\mathcal{F}} &= (!a \wedge !b) \to \bot
\end{aligned}
$$

For all $C \subseteq E$:

$$C \in \mathcal{F} \iff [\mathcal{E}]_{\mathcal{F}}, !C \vdash_{\mathsf{PCL}} C \text{ and } [\mathcal{E}]_{\mathcal{F}}, !C \nvdash_{\mathsf{PCL}} \bot$$

Being is configuration can be decided via logic

PCL vs LPN

The idea: associate to a Horn PCL theory a Lending Petri Nets, in a compositional fashion

PCL vs LPN

Let Δ a Horn PCL theory (formulas are of the kind $X \circ a$ with $\circ \in \{\vdash, \Vdash\}$ and X is a conjunction of atoms)

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Define a LPN $\langle S, T, F, L, \ell, m \rangle$ as follows

PCL vs LPN

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$$
\begin{array}{lcl}
T & = & \{(X, a, \rightarrow) \mid X \rightarrow a \in \Delta\} \cup \{(X, a, \twoheadrightarrow) \mid X \twoheadrightarrow a \in \Delta\} \\
S & = & \mathcal{L}(\Delta) \times (T \cup \{*\}) \text{ where } \mathcal{L}(\Delta) \text{ are the propositional letters in } \Delta \\
F(s,t) & = & \begin{cases} 1 & \text{if } \big(s = (a,*) \wedge t = (X, a, -)\big) \bigvee \big(s = (a,t) \wedge t = (\{a\} \cup X, c, \rightarrow)\big) \\ 0 & \text{otherwise} \end{cases} \\
F(t,s) & = & \begin{cases} 1 & \text{if } s = (a, t') \wedge t = (X, a, -) \wedge t' \neq * \\ 0 & \text{otherwise} \end{cases} \\
L(s,t) & = & \begin{cases} 1 & \text{if } s = (a,t) \wedge t = (\{a\} \cup X, c, \twoheadrightarrow) \\ 0 & \text{otherwise} \end{cases} \\
\ell(x) & = & \begin{cases} a & \text{if } x = (a,t) \in S \text{ or } x = (X, a, -) \in T \\ \bot & \text{otherwise} \end{cases} \\
m_0(s) & = & \text{if } s = (a,*) \text{ then } 1 \text{ else } 0
\end{array}
$$

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Consider the theory $\Delta = \{b \twoheadrightarrow a, a \rightarrow b\}$ where $\mathcal{L}(\Delta) = \{a, b\}$

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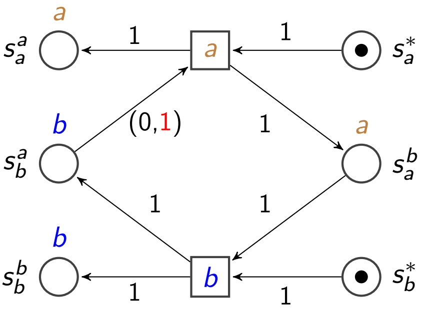
The associated LPN $\mathcal{P}(\Delta)$ is



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Given a PCL theory Δ and the associated LPN $\mathcal{P}(\Delta)$

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- $\overline{m} = \{a \in \mathcal{L} \mid m((a, *)) = 0\}$

  (the transitions that have been executed)

- $\Omega(m) = \{\ell(s) \mid m(s) < 0\}$

  (the tokens lent and not yet given back)

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Provability in PCL can be related with reachability of honored markings

$$\Delta \vdash X \iff \exists m \in Mk(\mathcal{P}(\Delta)).\ X \subseteq \overline{m} \wedge \Omega(m) = \emptyset$$

Labeling and interface in LPN

labels on places in a lending Petri net can be seen as the interface with the outside

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we can use them to compose LPNs

- $out(N) = \{s \in S \mid \ell(s) \neq \perp \text{ and } s^{\bullet} \cup s^{\circledcirc} = \emptyset\}$
  labeled places are output places if they only receive tokens
- $in(N) = \{s \in S \mid \ell(s) \neq \perp \text{ and } s^{\bullet} \cup s^{\circledcirc} \neq \emptyset\}$
  labeled places are input places if they may give away or lend tokens

Composing LPN

The nets must be disjoint (different places and different transitions) but may have labels in common

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Juxtapose the two net removing some output places and connecting transitions to input places with the same labels

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Juxtapose the two net removing some output places and connecting transitions to input places with the same labels

this operation is quite similar to the one on open nets

and the composition is commutative and associative

Formally...

Consider $N = \langle S, T, F, L, \ell, m_0 \rangle$ and $N' = \langle S', T', F', L', \ell', m'_0 \rangle$ such that $S \cap S' = \emptyset$ and $T \cap T' = \emptyset$ ... plus some other conditions on labeling

Formally...

The composition is $\langle \hat{S}, T \cup T', \hat{F}, \hat{\ell}, \hat{m}_0 \rangle$ where:

$$
\begin{array}{rcl}
\hat{S} & = & (S \cup S') \setminus (\mathbb{S} \cup \mathbb{S}'), \ \ \textit{where} \\
 & & \mathbb{S} \ = \ \{ s \in out(N) \mid \ell(s) \in \ell'(in(N')) \} \ \text{ and} \\
 & & \mathbb{S}' \ = \ \{ s \in out(N') \mid \ell'(s) \in \ell(in(N)) \} \\
\hat{F}(s,t) & = & \begin{cases} F(s,t) & \text{if } s \in S \text{ and } t \in T \\ F'(s,t) & \text{if } s \in S' \text{ and } t \in T' \\ 0 & \text{otherwise} \end{cases} \\
\hat{F}(t,s) & = & \begin{cases} F(t,s) & \text{if } s \in S \text{ and } t \in T \\ F'(t,s) & \text{if } s \in S' \text{ and } t \in T' \\ 1 & \text{if } t \in T \text{ and } s \in S' \text{ and } \ell(t) = \ell'(s) \\ & \text{or if } t \in T' \text{ and } s \in S \text{ and } \ell'(t) = \ell(s) \\ 0 & \text{otherwise} \end{cases}
\end{array}
$$

and the $\hat{L}$, $\hat{\ell}$ and $\hat{m}_0$ are determined componentwise

Composition

Output places are red and input places are green

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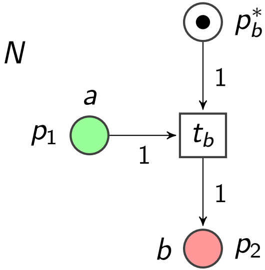

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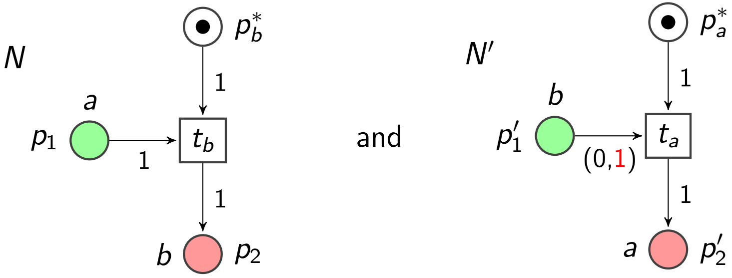

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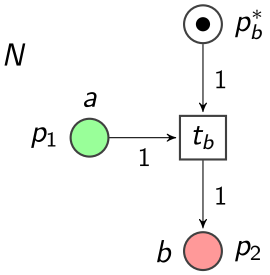
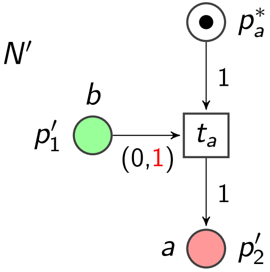
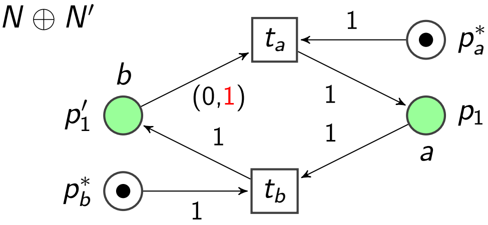


and





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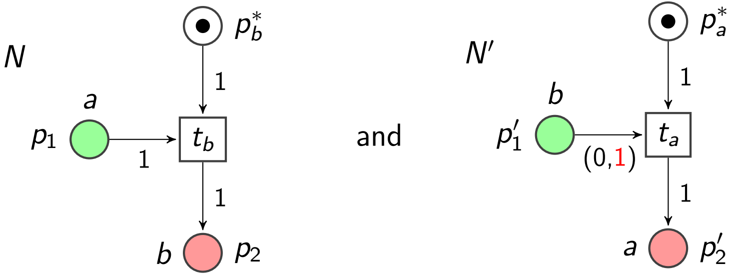

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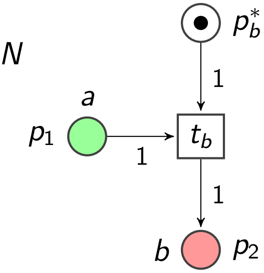

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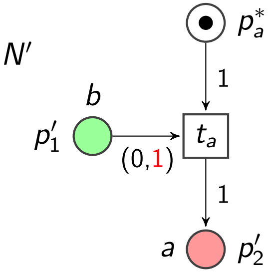
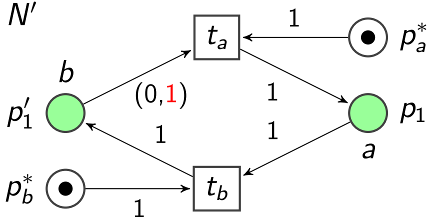


N is $\mathcal{P}(\{a \to b\})$ and N' is $\mathcal{P}(\{b \twoheadrightarrow a\})$

Composition

$N \oplus N'$



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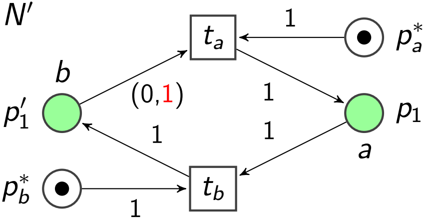

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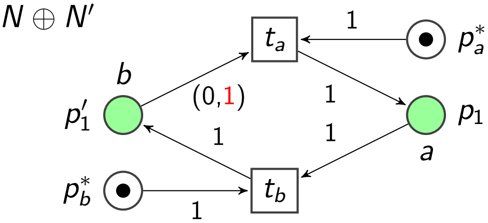

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$$\mathcal{P}(\Delta) \oplus \mathcal{P}(\Delta') \sim \mathcal{P}(\Delta, \Delta')$$

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we believe these models are more adherent to reality

Some conclusions

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- logical approach: ensuring agreement among various players (properties as formulae)

there are clear relations among these models

Some papers

- Bartoletti & Zunino: A calculus of contracting processes (LICS'10)
- Bartoletti & Tuosto & Zunino: Contract oriented computing in CO_2 (SACS, 2012)
- Bartoletti & Cimoli & Pinna & Zunino: An event based model for contracts (Places 2012)
- Bartoletti & Cimoli & Pinna & Zunino: Circular causality in event structures (FI, 2014)
- Bartoletti & Cimoli & Pinna: Lending Petri Nets and Contracts (FSEN '13)
- Bartoletti & Cimoli & Pinna: Lending Petri (SCICO)